

# Tunneling into Extra Dimension and High-Energy Violation of Lorentz Invariance

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## Abstract

We consider a class of models with infinite extra dimension, where bulk space does not possess  $SO(1,3)$  invariance, but Lorentz invariance emerges as an approximate symmetry of the low-energy effective theory. In these models, the maximum attainable speeds of the graviton, gauge bosons and scalar particles are automatically equal to each other and smaller than the maximum speed in the bulk. Additional fine-tuning is needed in order to assure that the maximum attainable speed of fermions takes the same value. A peculiar feature of our scenario is that there are no truly localized modes. All four-dimensional particles are resonances with finite widths. The latter depends on the *energy* of the particle and is naturally small at low energies.

## 1 Introduction and summary

Physics with extra dimensions has attracted considerable interest in the last few years. This recent activity is mainly related to the idea that large (or even infinite) extra dimensions are possible if matter fields [1, 2, 3] and gravity [4] are localized on a three-brane embedded in the bulk space of higher dimension (see, [5] for earlier papers where the idea of large extra dimensions was discussed). Hopefully, this approach may provide new insights into the cosmological constant [6, 7, 8] and hierarchy [3, 9] problems. Another advantage of the scenario with infinite extra dimensions is that it provides a framework for consistent treatment of those effects which are difficult to incorporate into conventional four-dimensional field theories. Examples of such effects are, e.g., modification of Newton's law at ultra-large scales [10], electric charge non-conservation [11] and mass generation for gauge bosons without the Higgs particle [12].

One more effect, which is hard to incorporate in conventional quantum field theories, is violation of Lorentz invariance at high energies. Certainly, Lorentz invariance is broken in our world by the cosmological expansion of the Universe. However, this effect is related to the dynamics at low energies and reveals itself only at ultra-large distances of the order of the Hubble scale. On the other hand, one may imagine that Lorentz invariance is broken by high-energy effects in complete theory incorporating quantum gravity. This idea dates back at least to early 80's [13, 14] and since that time both laboratory (see, e.g., [15] for recent review) and astroparticle (see, e.g., [16] for recent discussion) consequences of high-energy violation of Lorentz invariance we elaborated.

The resulting phenomenology can be addressed by introducing about 50 Lorentz-violating parameters in the Lagrangian of the Standard Model [17, 18], if one restricts oneself to renormalizable CPT even interactions only. The typical experimental limit on the dimensionless parameters in this approach is at the level of  $10^{-20}$  while some Lorentz violating parameters are limited at the level of  $10^{-30}$  [19].

Consequently, it is of interest to construct a consistent framework capable of predicting possible relations between various Lorentz-violating parameters and explaining their smallness. Scenarios with extra dimensions are natural candidates for such a framework. Another motivation to consider violation of Lorentz invariance in the context of models with extra dimensions comes from cosmology of these models. Namely, the standard assumption is that Lorentz invariance of low-energy effective theory requires bulk space with  $SO(1, 3)$  isometry. This gives rise to an analogue of the flatness problem in multidimensional cosmology, which may be even more severe than the flatness problem in the conventional cosmology [20]. The results of this paper indicate that  $SO(1, 3)$  isometry of the bulk space may be not necessary.

A mechanism to obtain a low-energy effective theory with approximate Lorentz invariance from extra dimensions has been recently suggested in Ref. [21] (see Refs. [22, 23, 20] for earlier works in this direction) where it has been proposed that this scenario may help to overcome difficulties of adjustment mechanisms aimed to solve the cosmological constant problem. The basic idea is to consider an asymmetric generalization of the Randall–Sundrum metric, with different warp factors for time and space coordinates,

$$ds^2 = a^2(z)dt^2 - b^2(z)d\mathbf{x}_i^2 - dz^2 . \quad (1)$$

Explicit constructions of this metric in Ref. [21] involve (charged) black hole solutions in the  $\text{AdS}_5$  space. It has been assumed that all matter and gauge fields reside on the brane located at  $z = 0$ , and, as a result, the only violation of Lorentz invariance comes from the gravitational sector. Approximate four-dimensional gravity emerges at low energies due to the presence of the localized graviton zero mode in the background (1). This mode has been found perturbatively, in the case of small difference between  $a(z)$  and  $b(z)$ . The complete structure of the low-energy effective gravitational action remains an open issue.

In this paper we concentrate on a different scenario, namely that matter and gauge fields, as well as gravitons, are localized modes of the bulk quantum fields in the background metric of the type (1). We show that violation of Lorentz invariance may reveal itself through a rather peculiar effect at low energies — metastability of all particles. This effect takes place if the extra coordinate  $z$  is non-compact and if the ratio of warp factors for time and space coordinates  $a(z)/b(z)$  tends to zero as  $z$  tends to infinity<sup>1</sup>. The reason for this effect is that in this case there are no truly localized modes. Instead, four-dimensional particles are described by narrow resonances in the continuum spectrum of the bulk modes. These quasilocalized modes are metastable states that decay into the continuum modes. The rate of the decay depends on the *energy* of the particle and is naturally suppressed at low energies.

From the point of view of a four-dimensional observer, this type of decays shows up as literal disappearance of the particles. This situation is similar to the metastability of massive particles in the conventional Randall–Sundrum background studied in Ref. [24]. We will closely follow Ref. [24] both in spirit and in some of the technicalities.

This Lorentz-violating effect is related to the presence of the continuum of bulk modes in models with infinite extra dimensions. It is difficult to incorporate it in the conventional approach [18] based on the notion of the low-energy effective action. A natural interpretation may be given in the holographic language, where disappearance of a particle may be interpreted as a decay into conformal matter, due to direct (Lorentz violating in our case) couplings between observed fields and conformal sector, which corresponds to the bulk modes in the AdS/CFT correspondence.

The structure of this paper is as follows. In Section 2 we describe a par-

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<sup>1</sup>Note that both of these properties are absent in the setup of Ref. [21].

ticular class of metrics of the type (1) with  $b(z) = \text{const}$ . To construct these metrics as solutions to the Einstein equations we invoke an antisymmetric two-form field interacting with gravity. These solutions may involve an arbitrary number (including zero) of extra compact warped dimensions. These extra dimensions are needed for the localization of bosons in the background metric with  $b(z) = \text{const}$ , their presence is not a necessary feature of more general Lorentz violating models with quasistable particles.

In Section 3 we describe tunneling into extra dimensions in the setup of Section 2, by using, as a specific example, fermions localized on the brane by Yukawa-type interactions. We first find the relation between energy and spatial momentum of the quasilocalized fermion mode. This dispersion relation is the same as in special relativity, but with the speed of massless fermions depending on the value of the Yukawa coupling between fermions and scalar field responsible for localization. We then calculate the life-time of the quasilocalized fermion and show that it is large at low energies. Both the speed of the quasilocalized fermion and its life-time do not depend on the number of the extra compact dimensions. In particular, the effect of quasilocalization exists in the absence of extra compact dimensions.

In the end of Section 3 we present an argument showing that only *quasilocalized* modes can be present in the background of the general type (1) with the described above property ( $a(z)/b(z) \rightarrow 0$  as  $z \rightarrow \infty$ ). Thus, with this type of background metric, tunneling into extra dimensions is a general property of all particles having bulk modes.

In Section 4 we consider quasilocalized scalar, vector and transverse graviton field, calculate their dispersion relations and widths, using solutions constructed in Section 2 as concrete examples of Lorentz violating metrics. We discuss the fine-tuning conditions needed to make the maximum attainable speed of fermions being equal to the speed of photons.

In Section 5 we briefly review phenomenological constraints on our scenario following from cosmic ray physics and searches for forbidden decays, and present our conclusions.

## 2 Example of the Lorentz violating metrics

In our explicit calculations which follow, we use asymmetrically warped metric of the form

$$ds^2 = e^{-2k|z|} \left( dt^2 - \sum_{a=1}^n (d\theta^a)^2 \right) - d\mathbf{x}_i^2 - dz^2 \quad (2)$$

Here  $t$  and  $\mathbf{x}_i$  ( $i = 1, 2, 3$ ) are the usual time and space coordinates and  $z$  is a coordinate along an infinite extra dimension. The coordinates  $\theta^a \in [0, 2\pi R_a]$  parameterize internal compact space which we take in the form of  $n$ -dimensional torus for the sake of simplicity (more generally, it can be an arbitrary compact Ricci-flat manifold). In what follows we assume that the radii of compact extra dimensions  $R_a$  are smaller than all distance scales of interest and take all fields to be constant along the  $\theta^a$ -directions.

Let us show how this metric can be obtained as a solution of field equations. Consider the following action involving  $(5+n)$ -dimensional metric  $g_{AB}$  and antisymmetric two-form field  $B_{AB}$ ,

$$S = \int d^{5+n}x \sqrt{|g|} \left( -\frac{M^{n+3}}{2} R - \Lambda + \frac{1}{4} H_{ABC} H^{ABC} \right). \quad (3)$$

Here  $M$  and  $\Lambda$  are bulk Planck mass and cosmological constant and  $H_{ABC}$  is the field strength tensor for two-form field  $B_{AB}$ ,

$$H_{ABC} = \partial_A B_{BC} + \partial_C B_{AB} + \partial_B B_{CA}.$$

In order to obtain the solution (2) we consider bulk space filled with the following constant background “magnetic” field

$$H_{ijk} = H \epsilon_{ijk}. \quad (4)$$

The non-vanishing components of the energy-momentum tensor corresponding to the magnetic field (4) in the background (2) are

$$T_{00} = -T_{\theta_a \theta_a} = \frac{3}{2} H^2 e^{-2k|z|}, \quad (5)$$

$$-T_{ii} = T_{zz} = -\frac{3}{2} H^2. \quad (6)$$

In addition we assume that there is a source brane located at  $z = 0$  with the following non-vanishing components of the energy-momentum tensor

$$\tau_{00} = -\tau_{\theta_a\theta_a} = \sigma_0\delta(z), \quad -\tau_{ii} = \sigma_x\delta(z). \quad (7)$$

The field equations for the two-form field are satisfied by the Ansatz (2) and (4). The Einstein equations lead to the following set of fine-tuning conditions,

$$2\Lambda = -(n+1)^2 k^2 M^{n+3}, \quad H^2 = (n+1)k^2 M^{n+3}, \quad (8)$$

$$\sigma_0 = 2knM^{n+3}, \quad \sigma_x = 2k(n+1)M^{n+3}. \quad (9)$$

This fine-tuning is somewhat similar to the fine-tuning inherent in the original Randall–Sundrum proposal.

A few comments are in order here. First, our solution may be considered as a limiting case of the hedgehog solutions with p-form fields described in Ref. [25]. The interpretation is quite different, however — dimensions which were treated as small and compact in Ref. [25] become ordinary infinite (or cosmologically large) spatial dimensions in our setup. On the other hand, small compact extra dimensions  $\theta^a$  in the metric (2) correspond to the usual spatial coordinates in the setup of Ref. [25].

Second, the energy-momentum tensor of the brane involved in our construction violates the positive energy condition, as is clear from Eq. (9) (the coefficient  $\omega$  in the equation of state  $p = \omega\rho$  is smaller than  $-1$  for the  $\mathbf{x}$ -component of pressure). Presumably, this will not lead to an instability as long as  $\mathbb{Z}_2$  orbifold symmetry  $z \rightarrow -z$  is imposed in analogy to the first Randall–Sundrum model [9]. It is worth noting that the same type of exotic sources was involved in the construction of Lorentz-violating setups in Ref. [21]. It remains to be understood whether exotic matter violating positive energy condition is a necessary ingredient of scenarios with approximate low-energy Lorentz invariance and localized gravity.

We note finally that, as pointed out in Ref. [26], metric of the type (2) has an orbifold singularity at  $z = \infty$  in the presence of warped extra dimensions  $\theta^a$ . We need these extra compact dimensions in order to obtain quasilocalized bosonic fields (see Section 4). However, the main effect studied in our paper (tunneling into extra dimensions in the geometry of the type (1)) is not related to this singularity as will be demonstrated in the next Section. This effect is present in a more general case of non-constant  $b(z)$  as well. In that case one need not add extra compact dimensions, leading to orbifold

singularity, because graviton and scalar particles are localized by pure gravity, and gauge field may be localized by some other mechanism (e.g., Dvali–Shifman mechanism [27]). Finally, as shown in Ref. [26], one of possible resolutions of this singularity in the context of string theory preserves the key feature of the Randall–Sundrum model relevant to the tunneling into extra dimension — a continuous spectrum of bulk modes starting from zero.

### 3 Tunneling into extra dimension

As the first example of tunneling into extra dimension let us consider quasilocalized fermions. For concreteness, in explicit calculations we make use of the metrics constructed in Section 2. In the end of this Section we show that tunneling into extra dimensions is a general feature of all particles having bulk modes, provided the Lorentz violating metrics (1) have the properties described in Introduction.

In the background of the type (2), fermionic fields are not localized on the brane by the gravitational interactions only, for the same reason as in the original Randall–Sundrum scenario [28] (see also [29]). Hence, one invokes the localization mechanism of Refs. [30, 1]. The simplest setup is as follows. One considers a domain wall formed by some scalar field  $\phi_k$ . This scalar field has a double-well potential with two degenerate vacua at  $\phi_k = \pm v$ ; the domain wall separates the region  $\phi_k = -v$  at  $z < 0$  from the region  $\phi_k = v$  at  $z > 0$ . In flat space, a fermionic field which has a Yukawa coupling to the scalar,  $g\phi_k\bar{\Psi}\Psi$ , has an exact zero mode in the domain wall background. This zero mode is topological and its existence does not depend on the details of the profile of the scalar field across the wall. Therefore, it exists also for infinitely thin wall,

$$\phi_k(z) = v\text{sgn}(z) ,$$

which is the case we consider in what follows.

The effective action for fermionic fields which are constant along the compact extra dimensions reads (cf. Ref. [29])

$$S = \int dz d^4x \sqrt{|g|} \bar{\Psi} \left( i\gamma^\alpha \nabla_\alpha - \frac{ikn \text{sgn}(z)}{2} \gamma^z + g\phi_k \right) \Psi, \quad (10)$$

where  $\nabla_\alpha$  is the spinor covariant derivative with respect to the metric (2) and  $\alpha = 0, i, z$ . For the sake of simplicity we limit ourselves to the case

of massless fermion. The treatment of massive fermions would require the introduction of a fermionic doublet in the bulk theory and would make our formulae more complicated without changing physics (cf. Ref. [24]). After the change of variables

$$\Psi = e^{k(n+1)|z|/2} \tilde{\Psi}$$

one obtains the following Dirac equation for the 4-spinor  $\tilde{\Psi}$ ,

$$\left[ E e^{k|z|} \gamma^0 + \gamma^i p_i + \gamma^5 \partial_z - g \phi_k(z) \right] \tilde{\Psi} = 0. \quad (11)$$

which is valid for any number of extra compact dimensions (and in their absence as well). In order to see that there exists a quasilocalized resonance, let us find the complex eigenvalue at which there exists a solution to Eq. (11) with the radiation boundary conditions imposed at  $z \rightarrow \pm\infty$ . It is convenient to choose the following basis for  $\gamma$ -matrices,

$$\begin{aligned} \gamma^0 &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma^1 = \begin{pmatrix} i\sigma_2 & 0 \\ 0 & -i\sigma_2 \end{pmatrix}, \quad \gamma^2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \\ \gamma^3 &= \begin{pmatrix} i\sigma_3 & 0 \\ 0 & -i\sigma_3 \end{pmatrix}, \quad \gamma^5 = \begin{pmatrix} -\sigma_1 & 0 \\ 0 & \sigma_1 \end{pmatrix} \end{aligned}$$

and separate the spinor  $\tilde{\Psi}$  into the up and down components

$$\tilde{\Psi} = \begin{pmatrix} \psi \\ \chi \end{pmatrix}.$$

Furthermore, we will make use of invariance of our metric under spatial rotations and consider a particle moving in the  $\mathbf{x}^3$ -direction, *i.e.* choose the spatial momentum in the form

$$\mathbf{p} = (0, 0, p)$$

In terms of two-component spinors  $\psi$  and  $\chi$ , Eq. (11) translates into a set of coupled equations

$$E e^{k|z|} \chi + i p \sigma_3 \psi - \sigma_1 \partial_z \psi - g \phi_k \psi = 0, \quad (12)$$

$$E e^{k|z|} \psi - i p \sigma_3 \chi + \sigma_1 \partial_z \chi - g \phi_k \chi = 0. \quad (13)$$

After eliminating  $\psi$  one obtains the second order equation for  $\chi$ ,

$$\left[ E^2 e^{2k|z|} + \partial_z^2 - k \operatorname{sgn}(z) (\partial_z - (p \sigma_2 + g \phi_k \sigma_1)) - g \phi'_k \sigma_1 - (g \phi_k^2 + p^2) \right] \chi = 0. \quad (14)$$



The differential operator in the left hand side of Eq. (14) coincides with the differential operator entering the second order equation for massive fermions (with mass equal to  $p$ ) in the Randall–Sundrum background; the latter has been considered in Ref. [24]. Consequently, we may make use of the result of that work and find that there is a complex eigenvalue determined by the following equation at  $p \ll gv$ ,

$$\frac{H_{M/k-1/2}^{(1)}(E/k)}{H_{M/k+1/2}^{(1)}(E/k)} = \frac{p}{2gv} ,$$

where  $M$  is defined in such a way that

$$gv + ip = Me^{i\alpha}$$

with real  $M$  and  $\alpha$ . We are interested in the case of small energies and momenta  $E, p \ll k, gv$ . In this case one expands the Bessel functions at small values of the argument and finds

$$E = E_0 - \frac{i\Gamma}{2}$$

with

$$E_0 = \left(1 - \frac{k}{2gv}\right) p \quad (15)$$

and

$$\Gamma = \frac{2\pi E_0}{[\Gamma(gv/k + 1/2)]^2} \left(\frac{E_0}{2k}\right)^{2gv/k-1} . \quad (16)$$

Equations (15) and (16) describe a narrow resonance at low energies, provided

$$\frac{gv}{k} > \frac{1}{2}$$

This resonance can be interpreted as a four dimensional metastable particle. The  $\chi$ -component of its wave function is determined by Eq. (14) and  $\psi$ -component of its wave-function may be found then from Eq. (13).

The speed of the quasilocalized massless fermion is equal to

$$c_f = 1 - \frac{k}{2gv}$$

as is clear from Eq. (15). This speed depends on the parameter  $k/2gv$  and is always smaller than the speed of a massless point-like particle tightly bound to the brane and moving along  $z = 0$  (the latter case corresponds to the limit  $gv \rightarrow \infty$ ). This does not necessarily imply that the maximum speed is not the same for different *quasilocalized* particles. As we will see below, quasilocalized scalar field, photon and graviton in the background (2) have equal speeds (smaller than the speed of a tightly bound massless particle). In order that the quasilocalized fermion has the same speed, additional fine-tuning of parameters is needed.

Another non-trivial manifestation of the violation of Lorentz invariance is non-zero imaginary part of the fermion energy, as given by Eq. (16). This imaginary part determines the probability for quasilocalized fermion to tunnel into extra dimension. The tunneling probability is small at low energies and grows with energy, as follows from Eq. (16). This effect is present in the absence of extra compact dimensions (at  $n = 0$ ) and, hence, is not related to the orbifold singularity at infinity.

Let us now present an argument indicating that metastability of all particles having bulk modes is a general property of the metrics of the type (1) with  $a(z)/b(z) \rightarrow 0$  as  $z \rightarrow \infty$ . Indeed, a mode equation for a bulk field  $\Phi$  in the background of this type may be presented in the following general form

$$[\Delta(z) + p^2 f(z) - E^2] \Phi(z) = 0, \quad (17)$$

where  $\Delta(z)$  is a positive semi-definite differential operator and the function  $f(z) = a^2(z)/b^2(z)$  vanishes at infinity. The key property of models with infinite extra dimensions, essential for our argument, is that the operator  $\Delta(z)$  has a continuous spectrum of energies starting from some  $E^2 = E_{min}^2$  at  $p = 0$ . This property may be considered as a definition of infinite extra dimension and means that the energy of the particle related to its motion along the  $z$ -direction is not quantized, at least starting from some large enough value. Note that in most of known models with infinite extra dimensions and localized gravity, the continuum spectrum starts from zero energy<sup>2</sup>  $E_{min} = 0$ . At non-zero  $p^2 > 0$ , the term  $p^2 f(z)$  in the left hand side of Eq. (17) vanishes at large  $z$  and thus it is irrelevant at infinity. Since the continuum eigenvalues are determined by the large  $z$  asymptotics only, Eq. (17) has the same

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<sup>2</sup>See, however, Refs. [31, 12]. We thank P. Tinyakov for pointing out the possibility of non-zero  $E_{min}$ .

continuum spectrum for all  $p^2$  including  $p^2 = 0$ . This implies that there are no truly localized modes with  $E^2 > E_{min}^2$  (there are no truly localized bound states embedded in the continuum) and all four-dimensional particles become unstable above this energy. A similar argument was presented in Ref. [24] for massive fields in the Randall–Sundrum background.

## 4 Quasilocalized bosons

As another example of tunneling into extra dimension, we consider propagation of the scalar field  $\phi$  with mass  $\mu$  in the background metric (2). The corresponding field equation has the following form

$$\left[-\partial_z^2 + (n+1)k\text{sgn}(z)\partial_z + \mu^2 + p^2 - E^2 e^{2k|z|}\right] \phi = 0, \quad (18)$$

where  $p$  is the spatial momentum and  $E$  is the energy. This equation is analogous to the field equation for massive scalar field in the Randall–Sundrum background. At a given value of  $p^2 > 0$ , there is a continuum spectrum of modes with arbitrary energies  $E^2 > 0$  and no localized modes. However, as in the case of massive fields in the Randall–Sundrum metric, there exists a resonance mode which describes quasilocalized particle. Indeed, let us show that the mode equation, Eq. (18), has a complex eigenvalue when the radiation boundary conditions are imposed at  $z \rightarrow \pm\infty$ . The solution to Eq. (18) which satisfies the radiation boundary conditions is

$$\phi(z) = c e^{k(n+1)|z|/2} H_\nu^{(1)}\left(\frac{E}{k} e^{k|z|}\right), \quad (19)$$

where  $H_\nu^{(1)}(x)$  is the Hankel function, the constant  $c$  is determined by the normalization condition and

$$\nu = \sqrt{\left(\frac{n+1}{2}\right)^2 + \left(\frac{p}{k}\right)^2 + \left(\frac{\mu}{k}\right)^2} \quad (20)$$

The first derivative of  $\phi(z)$  should be continuous at  $z = 0$ , as is clear from Eq. (18),

$$\partial_z \phi(+0) - \partial_z \phi(-0) = 0.$$

The latter condition implies the following dispersion relation

$$\frac{E}{k} \frac{H_{\nu-1}^{(1)}\left(\frac{E}{k}\right)}{H_{\nu}^{(1)}\left(\frac{E}{k}\right)} + \frac{n+1}{2} - \nu = 0. \quad (21)$$

Let us consider small energies and momenta  $\mu, E, p \ll k$ . Then expanding the Bessel function at small argument one finds

$$E = E_0 - \frac{i}{2}\Gamma \quad (22)$$

with

$$E_0^2 = \frac{n-1}{n+1}(p^2 + \mu^2) \quad (23)$$

and

$$\Gamma = \frac{2\pi E_0}{\Gamma(n/2)\Gamma(n/2-1)} \left(\frac{E_0}{2k}\right)^{n-1}. \quad (24)$$

We see that at  $n > 1$  (this is the case we consider in the rest of the Section) the width of the resonance is much smaller than  $E_0$ . As a result, this resonance can be interpreted as a four-dimensional metastable particle. This particle decays through tunneling into extra dimensions. Violation of Lorentz invariance reveals itself in the fact that the decay rate grows with the energy of the particle. The factor of  $(n-1)/(n+1)$  in the dispersion relation (23) implies that the maximum speed of this particle is smaller than the speed of a tightly bound massless particle moving along the brane at  $z = 0$ . In order that this speed be equal to the speed of the quasilocalized fermions, the following fine-tuning condition should be satisfied

$$1 - \frac{k}{2gv} = \sqrt{\frac{n-1}{n+1}}. \quad (25)$$

It would be desirable to obtain this fine-tuning condition as a consequence of some symmetry in the underlying theory. Another solution of this problem may be to construct a setup where fermions are localized by gravity only, like all other fields. See, e.g., Refs. [29, 32] for recent attempts in this direction. Alternatively, one can localize fermions on the additional spectator brane like in the Lykken–Randall scenario [33]. This brane should be located in the place where the maximum speed of tightly bound particles in  $\mathbf{x}_i$  directions

is the same as the speed of the quasilocalized bosonic modes. Finally, one can consider metrics of the type (1), i.e., more general than those described in Section 2. In that case the (approximate) equality between maximum attainable speeds of fermions and bosons may be attributed to the small difference between the warp factors  $a(z)$  and  $b(z)$ .

Let us now consider the propagation of a massless vector field  $V_A$  in the background (2). The field equation in curved space has the form

$$\partial_A \left( \sqrt{|g|} F^{AB} \right) = 0 , \quad (26)$$

where  $F_{AB} = \partial_A V_B - \partial_B V_A$ . To classify propagating modes of the vector field in the background (2), we find it convenient to work in the gauge

$$V_0 = 0 .$$

In this gauge the (0)-component of Eq. (26) takes the following form,

$$\partial_0 \left[ \partial_z \left( e^{-(n-1)k|z|} V_z \right) + e^{-(n-1)k|z|} \partial_i V_i \right] = 0 \quad (27)$$

The fact that the combination in the square bracket in the left hand side of Eq. (27) is constant in time makes it possible to use the residual gauge freedom to set this combination to zero,

$$\partial_z \left( e^{-(n-1)k|z|} V_z \right) + e^{-(n-1)k|z|} \partial_i V_i = 0 . \quad (28)$$

Then one can rewrite the (z)-component of Eq. (26) in the following form,

$$\left[ -\partial_z^2 + (n-1)k \operatorname{sgn}(z) \partial_z + p^2 - E^2 e^{2k|z|} + 2(n-1)k \delta(z) \right] V_z = 0 . \quad (29)$$

At zero energy and momentum, the field  $V_z = \text{const}$  ceases to be a solution of this equation because of the presence of the extra delta-functional term in the left hand side of Eq. (29), which is absent in Eq. (18). As a result, Eq. (29) does not admit localized modes at  $E = p = 0$  and one has no reason to expect the presence of narrow resonances at small but non-vanishing energy and momenta. Straightforward calculation analogous to one outlined in the beginning of this Section shows that such resonances are indeed absent. Consequently, the  $z$ -component of the vector field (or, equivalently, the longitudinal component of the photon, see Eq. (28)) does not have (quasi)localized modes in our setup.

Now, it is straightforward to check that transverse components  $V_i$  satisfy the same mode equation as the massless scalar field, Eq. (18) with  $\mu = 0$ . Consequently, a quasilocalized transverse photon with the same speed of light as the maximum speed of the scalar particle exists at  $n > 1$ .

Components  $V_{\theta^a}$  of the vector field do not couple to matter as long as there are no currents along the extra compact dimensions. However, for the sake of completeness, let us consider the propagation of these components as well. Taking  $(\theta^a)$ -component of Eq. (26) one obtains

$$\left[-\partial_z^2 + (n-1)k\text{sgn}(z)\partial_z + p^2 - E^2 e^{2k|z|}\right] V_{\theta^a} = 0. \quad (30)$$

Comparing this equation with the equation (18) for the scalar field we see that these components correspond to quasilocalized scalar particle with the maximum speed equal to  $\sqrt{(n-1)/(n-3)}$  at  $n > 3$ .

Let us now briefly discuss the quasilocalized graviton mode in our setup. The mode equation for transverse traceless gravitational waves may be obtained by considering metric perturbation of the form

$$\delta ds^2 = h(z)h_{ij}(\mathbf{x}, t)d\mathbf{x}^i d\mathbf{x}^j$$

with

$$\partial_i h_{ij} = \delta^{ij} h_{ij} = 0$$

It is straightforward to check that these perturbations decouple from perturbations of other components of the metric and of the two-form field  $B_{AB}$  and satisfy the same mode equation as the scalars, Eq. (18). This is the same situation as in the Randall–Sundrum model [34, 28] and as in the setup of Ref. [21]. Consequently, there exists a quasilocalized graviton in our setup with the same low-energy properties as the massless scalar boson. We will not study the excitations of other components of the metric and of the two-form  $B_{AB}$  here; this study would be necessary to understand the complete structure of the low-energy effective action for gravity in our setup and in the setup of Ref. [21]. We hope to return to this issue in future.

We would like to make only a couple of comments in this regard. The first one is that in Ref. [21], there were suggested two possible explanations of the difference between the speed of the localized graviton and the speed of a tightly bound massless particle moving along the brane at  $z = 0$ . The first possibility is that the effective action for gravity explicitly breaks the

4D Lorentz invariance at  $z = 0$ , as different coefficients for time and space coordinates in the kinetic term of the graviton are introduced. The second possibility is that the effective action violates the weak equivalence principle by the presence of some extra fields which couple differently to matter and gravity. This may force gravitational waves to propagate differently than the tightly localized particle.

To infer which of these two possibilities is actually realized, we note that the correct value of the maximum speed of the quasilocalized bosons can be obtained in the following empiric way. The scalar field action in the background (2) has the following form for the modes which are constant over compact coordinates  $\theta_a$

$$S_{sc} \propto \int dt d\mathbf{x} dz \left( e^{-(n-1)k|z|} \dot{\phi}^2 - e^{-(n+1)k|z|} (\partial_i \phi)^2 - e^{-(n+1)k|z|} \mu^2 \phi^2 \right). \quad (31)$$

Note that the field equation (18) admits a normalizable mode  $\phi(z) = \text{const}$  with zero energy when  $p = \mu = 0$ . To obtain the effective four-dimensional action for quasilocalized modes of low energy and momentum, one can plug a field  $\phi(t, \mathbf{x})$  independent of  $z$  in the action (31) and integrate over  $z$ . Then one will obtain precisely the dispersion relation (23) due to different coefficients in front of time and space derivatives in Eq. (31).

The fact that the above empirical way to obtain the low energy effective action for the scalar (and other bosons) gives the correct value for its speed due to the difference in the coefficients in front of time and space derivatives suggests that it is the effect of the first type that takes place (at least in our setup). Certainly, our observation does not exclude the possibility that the second effect may also be present.

The second comment is that one may worry that our setup suffers from a van Dam-Veltman-Zakharov (vDVZ) discontinuity as other theories with quasistable graviton [35]. In this regard it worth noting, that, as it was argued recently, this discontinuity is likely to be an artifact of the linear approximation [36]. Secondly, and more relevant for the class of model we consider, this discontinuity is due to the fact that quasilocalized graviton has extra polarization states in model discussed in [35]. In our models, graviton is strictly massless and may decay not because of non-zero mass but due to the presence of bulk modes with smaller propagation velocities. Consequently, one has no a priori reason to expect extra polarizations, which would give rise to the vDVZ discontinuity. Explicit analysis presented above demonstrates

that these polarizations are indeed absent in the photon case. The analysis of the linearized gravity in asymmetrically warped spaces which is necessary to address the same issue for a graviton is beyond the scope of this work.

## 5 Discussion

In this concluding section, let us first briefly discuss potential experimental constraints on the scenario suggested in this paper, by making use of the brane world solutions of Section 2 as a toy model. We assume that the fine-tuning condition (25) is satisfied and neglect possible non-universality in the maximum attainable speed of different quasilocalized particles which may appear due to quantum corrections. The latter issue requires a separate study. Then the limits on the parameters of our model are related to the instability of energetic particles and come from searches for forbidden decays and from physics of high-energy cosmic rays. In all our estimates we take the parameter  $k$  to be equal to the Planck mass  $M_{Pl} \sim 10^{19}$  GeV. Certainly, one can make all limits weaker by taking larger values of  $k$ . Also, one may expect that these constraints are weaker for more general metrics of the type given in Eq. (1), if the difference between warp factors  $a(z)$  and  $b(z)$  is not so drastic as in Eq. (2).

The first limit comes from the stability of electron. We estimate the width of electron against the decay  $e^- \rightarrow \text{nothing}$  by plugging  $E_0 = m_e$  in the expression (16) for the width of massless fermion and finding the parameter  $gv/k$  from the fine-tuning condition (25). As a result one finds that for two extra compact dimensions (the smallest number for which the localization of bosons takes place in our toy model), the life-time of electron is of order 100 years, which is certainly unacceptable. However, for three extra compact dimensions this life-time is  $9 \cdot 10^{25}$  years which is larger than the current experimental limit  $\tau_e > 4.2 \cdot 10^{24}$  years [37].

It is not quite obvious that Eq. (16) can be directly applied for estimating the proton life-time. Proton is a composite object and this fact may lead to an additional suppression of the decay rate. A naive application of Eq. (16) with  $E_0 = m_p$  excludes our setup with three extra compact directions, but for four extra compact dimensions gives the value  $\tau_p = 9.2 \cdot 10^{34}$  years which is much larger than the experimental limit on the decay  $p \rightarrow \text{nothing}$ ,  $\tau_p > 5 \cdot 10^{26}$  years [37].



Another set of limits comes from the high-energy cosmic ray data. Observation of 20 TeV photons from the distant blazar Markarian 501 [38] with redshift  $z \sim 0.3$  implies that the life-time of a photon of this energy cannot be much smaller than the age of the Universe (see Ref. [39] for a recent discussion of limits on the Lorentz-violating parameters coming from the observation of Markarian 501). By making use of the expression (24) for the life-time of bosons, we see that this observation excludes theories with the number of extra compact dimensions smaller than four, just like the proton decay.

The strongest limit may come from the observation of ultra-high energy cosmic rays. If the recently found correlation [40] of cosmic rays with energies  $E \sim 5 \cdot 10^{19}$  eV with BL Lacertae objects is confirmed by further data, then particles of this energy should be stable enough to travel as large distances as 600 Mpc. By making use of Eqs. (24) and (16) one finds that the number of extra compact dimensions in our toy model should be as large as 7, irrespectively of whether primary particles are fermions or bosons (assuming that one can use expressions (24) and (16) for composite objects like proton). This would mean that the total number of dimensions is equal to or greater than 12 and prevent embedding this brane world into string/M-theory. Note, however, that the life-times of particles strongly depend on the parameter  $k$ . Taking this parameter equal to  $5M_{Pl}$  will allow to have the total number of dimensions equal to 11, a value favored by M-theory.

To conclude, we constructed a setup with an infinite extra dimension where bulk space does not exhibit even approximate  $SO(1, 3)$  isometry but low-energy effective theory is approximately Lorentz invariant. This Lorentz invariance is automatic for scalar field, photon and graviton. Additional fine-tuning of parameters is needed to assure that the maximum attainable speed of fermions is the same. It worth noting that with this fine-tuning,  $SO(1, 3)$  is still strongly violated the bulk.

A peculiar property of our setup is that there are no truly localized modes — all four-dimensional particles are narrow resonances in the spectrum of bulk modes. These resonances have finite probabilities to tunnel into extra dimension. These probabilities depend on the energies of the particles.

In the (generalized) holographic interpretation where bulk modes are described as a hidden (quasi)conformal sector (see, e.g., Refs. [41, 34, 42, 43])

this type of decay may be attributed to the direct coupling

$$\phi \mathcal{O}_\phi$$

between quasilocalized fields  $\phi$  and operators  $\mathcal{O}_\phi$  from the conformal sector. This coupling may be Lorentz invariant like in the case of massive particles in the Randall – Sundrum background [24] or violating Lorentz invariance like in the examples of this paper. Tunneling into extra dimension is interpreted then as a decay into conformal matter. It worth noting that in this language, emergence of the Lorentz invariant sector at low energies, in a theory without Lorentz invariance at high energies, appears rather non-trivial.

Possibly, one can consider bulk metrics even without  $SO(3)$  symmetry under spatial rotations. For instance, one can take metric with time and one of the space coordinates warped and two other coordinates not warped. Presumably, one will be able to fine-tune parameters so that all maximum attainable speeds will be equal for different fields in this setup as well. An interesting property of these metrics is that the tunneling probability will depend not only on the energy of the particle, but also on the direction of its motion.

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